Games Seminar: Pushdown Parity Games

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22/02/2022

Plan for this talk

- Why Pushdown Games?

Plan for this talk

- Why Pushdown Games?
- What Pushdown Games?

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- Why Pushdown Games?
- What Pushdown Games?
- How Pushdown Games?

Types of games seen so far

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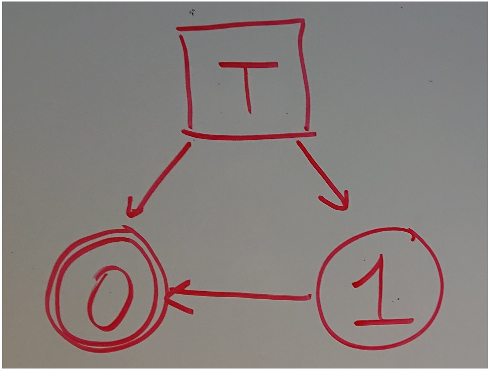
▶ Infinite games?

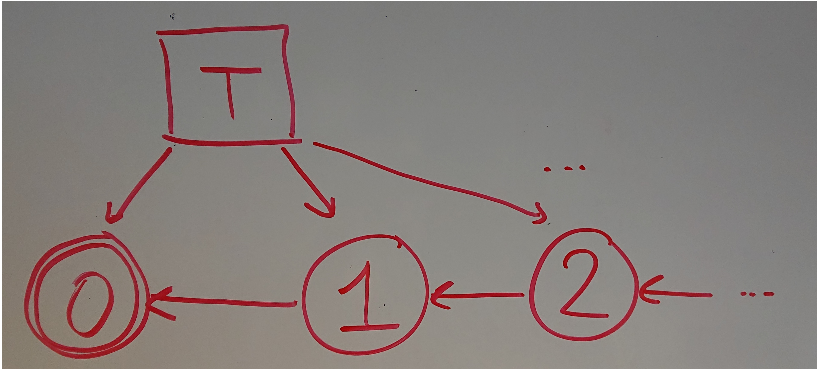
Motivation(s)

Synthesis for systems over $\mathbb{N}$ or $\mathbb{R}$, ...

Differences between finite/infinite games



Differences between finite/infinite games



Infinite games as inputs

- Need to give games as inputs for decision problems
- But cannot give them as infinite set of nodes/edges

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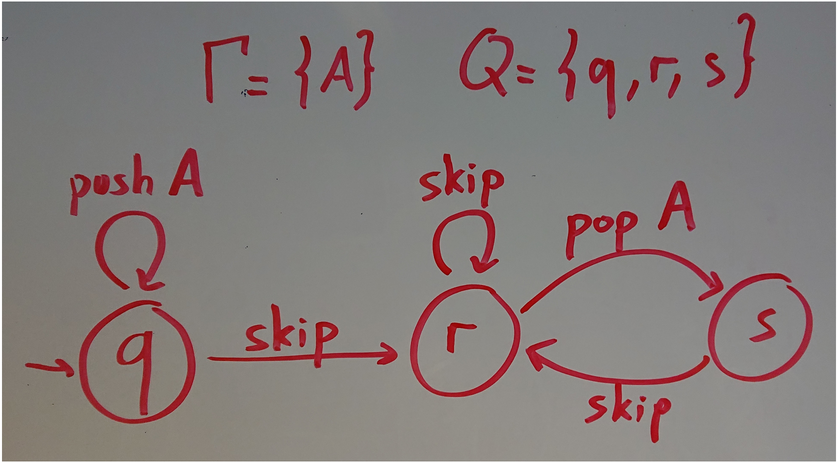
▶ Need finite representation!

Pushdown Processes



Pushdown Processes

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$\mathcal{P} = (\Gamma, Q, q_0, \delta : Q \times \Gamma \to Q \times \mathrm{Op})$
with $\mathrm{op} \in \mathrm{Op}$ of the form $\mathrm{push}\ A$ or $\mathrm{pop}\ A$ for $A \in \Gamma$.

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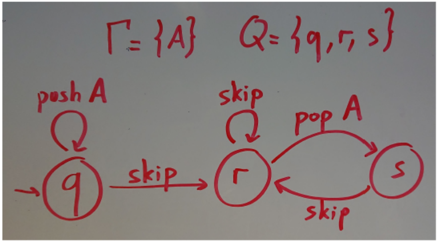
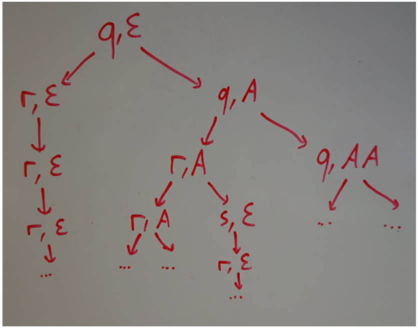
Configurations

Configuration of $\mathcal{P}$: $(q, \gamma) \in Q \times \Gamma^*$

Configuration tree





Pushdown Parity Games (PPG)

Game over tree of configurations

- $Q = Q_0 \uplus Q_1$ partition of states

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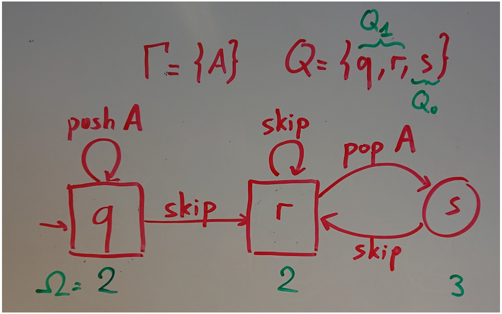

Reduction to finite parity game [I. Walukiewicz, CAV 96]

Goal

Reduce configuration $(q, \gamma A)$ into (q, A) + some information

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1. If A never popped $\Rightarrow \gamma$ irrelevant
2. If A popped $\Rightarrow$ only need to know the state after the pop

Main idea illustrated

On push, Player 0 guesses the set of possible states S following the corresponding pop

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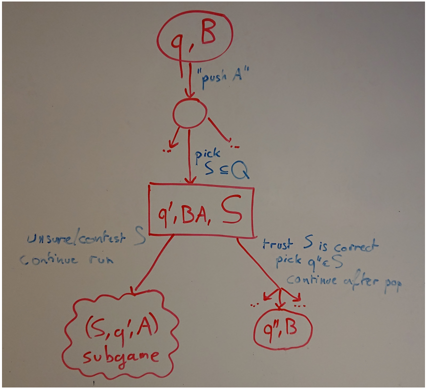
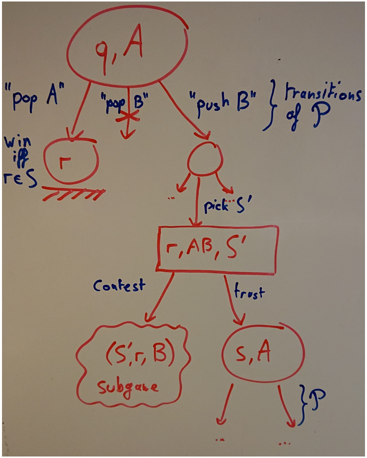
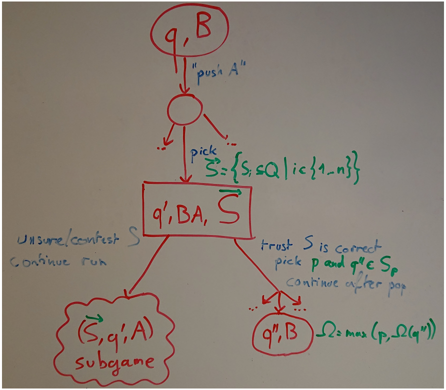


(S, q, A)-subgame


q, A
"pop A"
"pop B"
"push B"
transitions of P
win iff r∈S
r
pick S'
r, AB, S'
Contest
trust
(S', r, B) subgame
s, A
P

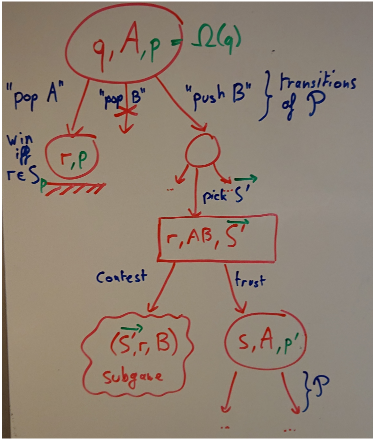
Again, with parity

On push, Player 0 guesses the sets of possible states (S_p) following the corresponding pop when p is the max priority seen inbetween



Subgame with parity

Need to keep track of max priority seen



"Sketch" of correctness

Lemma

Player 0 cannot win by guessing $\overrightarrow{S}$ wrong

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Correctness

Player 0 wins in the PPG iff Player 0 wins in the finite game

What about the complexity?

Each $(\overrightarrow{S}, q, A)$-subgame has size exponential in the size of $\mathcal{P}$ (due to guessing the $\overrightarrow{S'}$)

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Exponential reduction!

Conclusion and extensions

Main result

A PPG can be reduced to an exponential size (finite) parity game

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Extensions

- Multi Pushdown Games: not 1 but k stacks
- Higher Order Pushdown Games: multi-dimensional stacks

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Thanks!